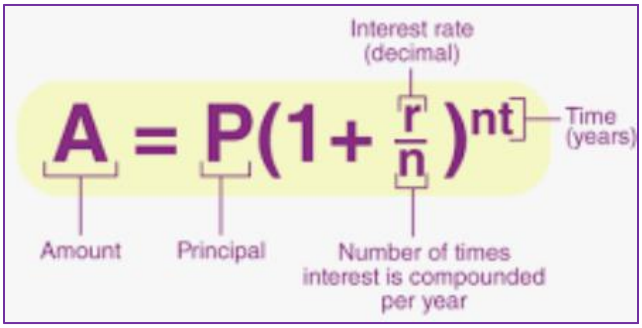
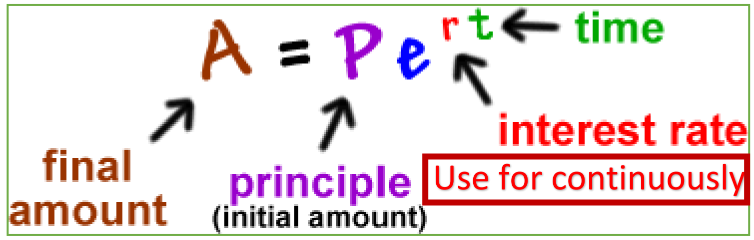
4.5 Application of Exp and Log Functions   MATH 1610   THOMPSON





1a) Find the amount that results from the given investment

$600 invested at 7% compounded quarterly after a period of 3 years use divide not fraction

After 3 years, the investment results in $____

Quarterly → n = 4 $600\left(1+\left(\frac{0.07}{4}\right)\right)^{\wedge(4\cdot 3)} \approx \$738.86$

*****calculator steps: 600(1+(.07/4))^(4·3)***

1b) Find the amount that results from the given investment

$300 invested at 12% compounded quarterly after a period of 4 ½ years 4.5

After 4 ½ years, the investment results in $____

Quarterly → n = 4 $300\left(1+\left(\frac{0.045}{4}\right)\right)^{\wedge(4\cdot 4.5)} \approx \$510.73$

*****calculator steps: 300(1+(0.12/4))^(4·4.5)***

1c) Find the amount that results from the given investment

$400 invested at 3% compounded daily after a period of 2 years

After 2 years, the investment results in $____

Daily → n = 365 $400\left(1+\left(\frac{0.03}{365}\right)\right)^{\wedge(365\cdot 2)} \approx \$424.73$

*****calculator steps: 400(1+(0.03/365))^(365·2)***

2) Find the amount that results from the given investment

$10 invested at 11% compounded continuously after a period of 2 years

After 2 years, the investment results in $____

continuously Pe^{rt} $10e^{(0.11\cdot 2)} \approx \$12.46$

3) Find the principal needed now to get the given amount, that is, find the present value

T<u>o get</u> $90 after 3¼ years at 9% compounded monthly   *monthly → n = 12*   *3¼ =3.25*

* <u>**to get use negative exponent**</u> $90\left(1+\left(\frac{0.09}{12}\right)\right)^{\wedge(-3.25\cdot 12)} \approx$ **$67.18**

**negative exponent means dividing*

4) Find the principal needed now to get the given amount, that is, find the present value

To get $60 after 2½ years at 4% compounded continuously.

* to get use negative exponent $60e^{(-0.04\cdot 2.5)}$

negative exponent means dividing

P ≈ $54.29

5) What rate of interest compounded annually is required to double an investment in 5 years?

$A = P\left(1+\frac{r}{n}\right)^{(nt)}$ *A = 2P (doubled) and n = 1*

$2P = P(1+r)^5$

Always put 2 on the left when anything doubles

$2 = (1+r)^5$ left side is always 2 when is doubled

$\sqrt[5]{2} = 1 + r$

$\sqrt[5]{2} - 1 = r$ r = 0.1487 = 14.87%

*****calculator steps 5 2nd ^ 2 enter – 1***

6) How long does it take for an investment to double in value if it is invested at 11% compounded monthly? Compounded continuously.

Monthly: $2 = \left(1+\frac{0.11}{12}\right)^{(12t)}$ take ln of both sides

$\ln 2 = \ln\left(1+\frac{0.11}{12}\right)^{(12t)}$ move 12t in front

$\ln 2 = 12t\ln\left(1+\frac{0.11}{12}\right)$ divide by everything but t

$t = \frac{\ln 2}{12\ln\left(1+\frac{0.11}{12}\right)} = 6.33$

*****calculator steps: ln2)÷(12ln(1+(0.11/12)))***

Continously: $2 = (e)^{(.0.11\cdot t)}$

$\ln 2 = 0.11t\ln e$

$t = \frac{\ln 2)}{0.11} = 6.30$

7) Jerome will be buying a used car for $11,000 in 4 years. How much money should he ask his parents for now so that, if he invests it at 8% compounded continuously, he will have enough to buy the car?

**negative exponent means dividing*

$11,000(e)^{(-.08\cdot 4)}$ **P≈ $7987.64**

8) The size of P of a certain insect population at time t (in days) obeys the function $P(t) = 300e^{0.06t}$

a) Determine the number of insects at t = 0 days. $300e^{0.06(0)} \approx$ **300**

b) What is the growth rate of the insect population? 6%

c) What is the population after 10 days? $300e^{0.06(10)} \approx$ **547 insects**

d) When will the insect population reach 510? $\frac{510}{300} = \frac{300e^{0.06(t)}}{300}$ *divide by 300*

$1.7 = e^{.06t}$

take ln of both sides $\frac{\ln 1.7}{0.06} = \frac{0.06t}{0.06}$

t = 8.8

e) When will the insect population double?

Always put 2 on the left when anything doubles

$2= e^{.06t}$

$\ln 2=.06t$

$\frac{\ln 2}{0.06} = t$

t = 11.6

9) Stronium 90 is a radioactive material that decays according to the function $A(t) = A_0e^{-0.0244t}$, where A_0 is the initial amount time t (in years). Assume that a scientist has a sample of 400 grams of strontium 90.

a) What is the decay rate of strontium 90? **-2.44%**

b) How much strontium 90 is left after 20 years? $400e^{-0.0244(20)} \approx$ **246**

c) When will only 300 grams of strontium 90 be left? $\frac{300}{400}=\frac{400e^{-0.0244t}}{400}$

$0.75=e^{-0.0244t}$

take ln of both sides $\frac{\ln .75}{-0.0244}= \frac{-0.0244t}{-0.0244}$ **t ≈ 11.8**

d) What is the half-life of strontium 90? **Always put 0.5 on the left for half-life**

$0.5 = e^{-0.0244t}$

take ln of both sides

$$\frac{\ln 0.5}{-0.0244} = \frac{-0.0244t}{-0.0244}$$

$t \approx 28.4$

10) **In a town whose population is 2700, a disease creates an epidemic. The number of people N infected days after the disease has begun is given by the function:** $N(t) = \frac{2700}{1+22.9e^{-0.7t}}$

a) How many are initially infected with the disease (t=0)?

$$\frac{2700}{(1+22.9e^{-0.7\cdot 0}} = 113$$

b) Find the number infected after 2 days, 5 days, 8 days, 12 days, and 16 days.

The number infected after 2 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 2}} = 406$

The number infected after 5 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 5}} = 1596$

The number infected after 8 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 8}} = 2489$

The number infected after 12 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 12}} = 2686$

The number infected after 16 days is $\frac{2700}{(1+22.9e^{-0.7\cdot 16}} = 2699$

As $t \to \infty$ n(t) = 2500, the number approaches 2500 but never actually reaches it.

11) The sound level, L, in decibels (db), is given by the formula

$L = 10\cdot\log(I \times 10^{12})$ db, where I is the intensity of the sound in watts per square meter. The sound level is 90db. What value of I gives sound of 90db?

$90 = 10\cdot\log(I \times 10^{12})$

$9 = \log(I \times 10^{12})$

$10^9 = I \times 10^{12}$

$I = \frac{10^9}{10^{12}} = .001$

12) The supply function and demand function for the sale of a certain type of DVD player are given by $S(p) = 140e^{0.004p}$ and $D(p) = 448e^{-0.002p}$, where S(p) is the number of DV players that the company is willing to sell at price p and D(p is the quantity that the public is willing to buy at price p. Find p such that D(p) = S(p). This is called the equilibrium price.

$448e^{-0.002p} = 140e^{0.004p}$ divide left by 140

$3.2e^{-0.002p} = e^{0.004p}$ take ln of both sides

$\ln(3.2e^{-0.002p}) = \ln e^{0.004p}$

$\ln 3.2 + \ln e^{-0.002p} = \ln e^{0.004p}$

$\ln 3.2 - 0.002p = 0.004p$

$\ln 3.2 = 0.006p$

$\frac{\ln 3.2}{0.006} = p$

\$193.86 = p

divide $\frac{448}{140}$= **3.2 ADD the exponents(drop negative) 0.004 + 0.002=**0.006**

Divide $\frac{\ln 3.2}{0.006}$ = **\$193.86**

EXTRA EXAMPLES

a) According to Newton's Law of Cooling, if a body with temperature T_1 is placed in surroundings with temperature T_0 different from that of T_1, the body will either cool or warm to temperature T after t minutes, where $T(t) = T_0 + (T_1 - T_0)e^{-kt}$.

A chilled jello salad with temperature 42°F is taken from a refrigerator and placed in a 68°F room. After 5 minutes, the temperature of the salad is 48°F. Use Newton's Law of Cooling to find the salad's temperature after 10 minutes.

48-68= -20 42-68 = -26

$$k = \frac{\ln(\frac{20}{26})}{5} = -0.05$$

After 10 minutes the jello salad will have a temperature of 53 °F.
(Round to the nearest integer.)

$$68 - 26e^{-0.05 \cdot 10} = 53$$ **ALWAYS ROUND UP**

b) According to Newton's Law of Cooling, if a body with temperature T_1 is placed in surroundings with temperature T_0 different from that of T_1, the body will either cool or warm to temperature T after t minutes, where $T(t) = T_0 + (T_1 - T_0)e^{-kt}$.

A chilled jello salad with temperature 49°F is taken from a refrigerator and placed in a 68°F room. After 15 minutes, the temperature of the salad is 55°F. Use Newton's Law of Cooling to find the salad's temperature after 20 minutes.

55-68= -13 49-68 = -19

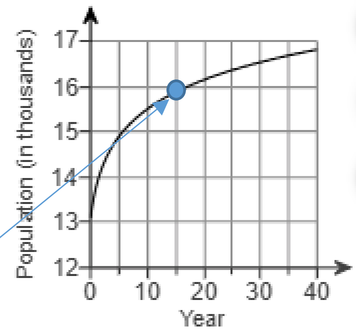
$$k = \frac{\ln(\frac{13}{19})}{15} = -0.025$$

After 20 minutes the jello salad will have a temperature of 57 °F.
(Round to the nearest integer.)

$$68 - 19e^{-0.025 \cdot 20} = 57$$ **ALWAYS ROUND UP**

c) The population of an animal in a national forest is modeled by the formula P = 13,100 + 1000 • **ln** (t + 1), where t is the number of years from the present.

(a) Use the formula to determine how many animals are now in the forest.
(b) Use the accompanying graph to estimate the number of years that it will take for the animal population to reach 16,000.
(c) Use the formula to determine the number of years that it will take for the animal population to reach 16,000.



(a) At present, there are 13,100 animals in the forest.

(b) It will take approximately 15 years for the animal population to reach 16,000.

(c) It will take approximately 17.2 years for the animal population to reach 16,000.
(Round to one decimal place as needed.)

16000 = 13100 + 1000 ln(t+1)
-13100
2900 = 1000 ln(t+1)
1000
2.9 = ln(t+1) put e on both sides
$e^{2.9)} - 1$ = t
17.2 = t

d) The sound level, L, in decibels (db), is given by the formula
L = 10·log(I × 10^{12}) db, where I is the intensity of the sound in watts per square meter. The sound level is 90db. What value of I gives sound of 90db?

90 = 10·log(I × 10^{12})
9 = log(I × 10^{12})
10^9 = I × 10^{12}

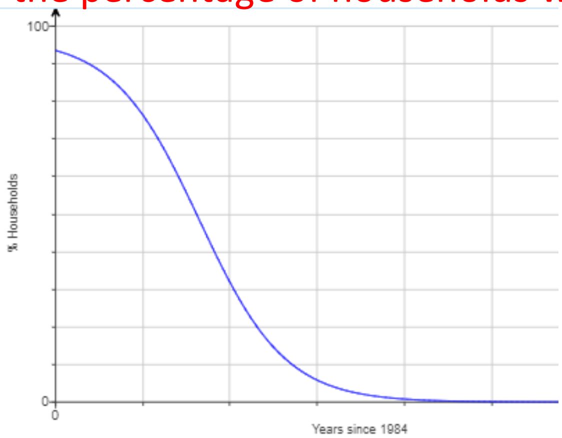
I = $\frac{10^9}{10^{12}}$ = .001

e) The logistics model $\frac{96.8181}{1+0.0351e^{0.203t}}$ represents the percentage of households that do not own a personal computer t years since 1984.

a) Evaluate an interpret P(0) take off e term $\frac{96.8181}{1+0.0351}$ =93.5

P(0) is the percentage of households without a personal computer in 1984

b) Graph



c) What percentage of households did not own a personal computer in 1996?

t = 1996-1984 = 12 plug in t=12 $\frac{96.8181}{1+0.0351e^{0.203\cdot 12}}$ =69.1

d) In what year will the percentage of households that do not own a personal computer reach 20%?

$$20 = \frac{96.8181}{1+0.0351e^{0.203t}}$$

$$20(1 + 0.0351e^{0.203t}) = 96.8181$$

$$20 + 0.702e^{0.203t} = 96.8181$$

-20

$$0.702e^{0.203t} = 76.8181$$

$$t = \frac{\ln\left(\frac{76.8181}{0.702}\right)}{0.203} = 23.1$$

23.1 + 1984 = **2007**

e) If Tanisha has $100 to invest at 8% per annum (a)compounded quarterly, how long will it be before she has $200? *quarterly → n = 4*

$200 = 100\left(1 + \frac{0.08}{4}\right)^{(4t)}$ *divide by 100 $2 = (1.02)^{4t}$ *then take ln of both sides*

$\rightarrow \frac{\ln 2}{4\ln 1.02} = \frac{(4t)\ln 1.02}{4\ln 1.02}$

divide right side to get t: $\left(\frac{\ln(2)}{(4\ln 1.02)}\right) \rightarrow t \approx 8.75$

(b) compounded continuously, how long will it be?

$200 = 100(e)^{(.08t)}$ *divide by 100 $2 = e^{0.08t}$ *then take ln of both sides →*

$\frac{\ln(2)}{.08} = \frac{.08t}{.08}$ lne = 1 so its cancelled

$t \approx 8.66$

f) What will a $210,000 house cost 5 years from now if the price appreciation for homes over the period averages 3% compounded annually?

$A = 210000\left(1 + \left(\frac{0.03}{1}\right)\right)^{(5)}$ A ≈ $243,447.56

g) Survey estimates the current average cost for college to be $30,490 per year.

a) If the average cost increases by 8.5%, what will the cost by 10 years from now?

$A = 30490\left(1 + \left(\frac{0.085}{1}\right)\right)^{(10)}$ A ≈ $68937.39

b) If a savings plan offers 7.6% compounded continuously, how much should be put in a plan now to pay one year of college 10 years from now?

$68937.39 = Pe^{(0.076 \cdot 10)}$ $68937.39e^{(-0.076 \cdot 10)}$ P ≈ $32239.70

**negative exponent means dividing*

h) Find the principal needed now to get the given amount, that is, find the present value

<u>To get</u> $200 after 4 years at 9% compounded quarterly.

$$\mathbf{200 = P\left(1+\frac{0.09}{4}\right)^{(4\cdot 4)}} \qquad \mathbf{200\left(1+\frac{0.09}{4}\right)^{(-4\cdot 4)}}$$

*** <u>to get use negative exponent</u>**

**negative exponent means dividing*

P ≈ $140.09

i) Find the principal needed now to get the given amount; that is, find the present value

To get $200 after 4 years at 4% compounded quarterly

$$\mathbf{200 = P\left(1+\left(\frac{0.04}{4}\right)\right)^{(4\cdot 4)} \rightarrow P = 200\left(1+\left(\frac{0.04}{4}\right)\right)^{(-4\cdot 4)}}$$

*** <u>to get</u> use negative exponent**

**negative exponent means dividing to find P*

P ≈ $170.56

j) Find the principal needed now to get the given amount; that is, find the present value.

To get $60 after $2\frac{1}{4}$ years at 5% compounded continuously

$$\mathbf{60 = Pe^{(0.05\cdot 2.25)} \rightarrow \quad 60e^{(-0.05\cdot 2.25)}}$$

*** <u>to get</u> use negative exponent**

**negative exponent means dividing*

P ≈ $53.62

k) Find the amount that results from the given investment.

$60 invested at 8% compounded continuously after a period of 4 years

$$\mathbf{60e^{(0.8\cdot 4)} \approx \$82.63}$$

l) The population of a colony of mosquitoes obeys the law of uninhibited growth. If there are 1000 mosquitoes initially and there are 1400 after 1 day, what is the size of the colony after 4 days? How long is it until there are 50,000 mosquitoes?

**k replaces r in the formula and find k first*

Find **rate (k)** first: $1400 = 1000e^{k(1)}$ → 4 days → $1000e^{(4 \cdot \ln 1.4)}$

divide by 1000 $1.4 = e^{k}$ **≈ 3842 mosquitoes**

$\ln 1.4 = k$

$50000 = 1000e^{\ln 1.7t}$

divide by 1000 $50 = e^{\ln 1.7t}$

take ln of both sides $\ln 50 = \ln 1.4t$ $t = \frac{\ln 50}{\ln 1.4}$ **t ≈ 11.6 days**

m) Jerome will be buying a used car for $10,000 in 2 years. How much money should he ask his parents for now so that, if he invests it at 9% compounded continuously, he will have enough to buy the car?

$\mathbf{10,000 = P(e)^{(.09 \cdot 2)}}$ **negative exponent means dividing*

$\mathbf{10,000(e)^{(-.09 \cdot 2)}}$ **P≈ $8352.70**

n) Find the amount that results from the given investment.

$500 invested at 9% compounded daily after a period of 3 years

Have to put all parenthesis $\mathbf{500\left(1 + \left(\frac{0.09}{365}\right)\right)^{(3 \cdot 365)} \approx \$654.96}$

**exponent also has to be in parenthesis after the ^ key*

o) How many years will it take for an initial investment of $10,000 to grow to $25,000? Assume a rate of interest of 9% compounded continuously.

$\mathbf{25,000 = 10,000(e)^{(.0.09 \cdot t)}}$

$\mathbf{2.5 = (e)^{(0.09t)}}$ $2.5 = e^{0.09t}$ *then take ln of both sides* →

$\mathbf{\ln(2.5) = 0.09t}$

$\frac{\ln(2.5)}{0.09} = \frac{0.09t}{0.09}$ **t ≈ 10.18**

p) The size P of a certain insect population at time t (in days) obeys the function $P(t) = 800\,e^{0.04t}$.

(a) Determine the number of insects at t = 0 days. $800e^{0.04(0)} \approx$ **800**

(b) What is the growth rate of the insect population? 4%

(c) What is the population after 10 days? $800e^{0.04(10)} \approx$ **1193 insects**

(d) When will the insect population reach 1200?

$1200 = 800e^{0.04(t)}$ *divide by 300*

$1.5 = e^{.04t} \rightarrow \frac{ln1.5}{.04} = \frac{.04}{.04}t$ **t = 8.8**

e) When will the insect population double? $2 = e^{.04t} \rightarrow \ln 2 = .04t$ **t = 11.6**

q) The half-life of carbon-14 is 5600 years. If a piece of charcoal made from the wood of a tree shows only 61% of the carbon-14 expected in living matter, when did the tree die?

Find rate (k) first: $\ln 0.5 = e^{5600k}$ $\quad .61 = e^{(\frac{ln0.5}{5600})t}$

$k = \frac{ln0.5}{5600}$ $\quad \ln .61 = \frac{ln0.5}{5600}t$ **multiply by reciprocal*

$\frac{(5600 \cdot ln0.61)}{ln0.5} = t \approx$ **3993**

r) Strontium 90 is a radioactive material that decays according to the function $A(t) = A_0\,e^{-0.0244t}$, where A_0 is the initial amount present and A is the amount present at time t (in years). Assume that a scientist has a sample of 400 grams of strontium 90.

a) What is the decay rate of strontium 90? -2.44%

b) How much strontium 90 is left after 10 years?

we just plug in 10 for t* $\quad 400e^{-0.0244(00)} \approx$ **313

c) **When will only 100 grams of strontium 90 be left?** $100=400e^{-0.0244t}$

300 is what we end with so we divide $\frac{100}{400} = .75$ $0.25=e^{-0.0244t}$

take ln of both sides $\ln .25 = -0.244t \rightarrow \frac{ln0.25}{-0.0244} = \frac{-0.0244t}{-0.0244} t$ **t ≈ 56.8**

d) **What is the half-life of strontium 90?**

$0.5=e^{-0.0244t}$

take ln of both sides $\ln .5 = -0.0244t$ $\frac{ln0.5}{-0.0244} = \frac{-0.0244t}{-0.0244} t$ **t ≈ 28.4**

s) If Tanisha has $ 100 to invest at 9% per annum compounded monthly, how long will it be before she has $ 250? If the compounding is continuous, how long will it be?

(a) monthly → *n = 12*

$$250 = 100\left(1+\frac{0.09}{12}\right)^{(12t)}$$ **divide by 100* $2.5 = (1.0075)^{4t}$ *then take ln of both sides*

$\rightarrow \frac{\ln 2.5}{(4\ln 1.0075)} = \frac{(4t)\ln 1.0075}{4\ln 1.0075}$

divide right side to get t: $\left(\frac{\ln(2)}{(4ln1.02)}\right) \rightarrow$ **t ≈ 10.22**

(b) compounded continuously, how long will it be?

$250 = 100(e)^{(.09t)}$ **divide by 100* $2.5 = e^{0.09t}$ *then take ln of both sides* →

$\frac{\ln(2.5)}{.09} = \frac{.09t}{.09}$

t ≈ 10.18